ACT Math Guide: Area, Perimeter, & Volume

Summit Math Lab

Introduction

These are the "Free Points" of the ACT. If you know the formula, you get the point. If you don't, you guess. The most common formula students forget is the **Trapezoid**, so pay special attention to Section 3.

This guide covers:

1. **Perimeter** (The border)
2. **Area of Polygons** (Triangles & Parallelograms)
3. **Area of a Trapezoid** (The "Average Base" Rule)
4. **Volume** (Prisms & Cylinders)

1. Perimeter (The Border)

Perimeter is the total distance around the outside of a shape.

- **Rule:** Add up all the sides.
- **Common Mistake:** If a shape is composite (two shapes stuck together), do **not** count the line in the middle. Only count the outside edges.

2. Area of Basic Polygons

Area is the space *inside* the shape.

A. Parallelograms (and Rectangles)

$$A = b \cdot h$$

Crucial: The height (h) must be **Perpendicular** (90°) to the base.
Note: If you are given a slanted side, that is NOT the height.

B. Triangles

$$A = \frac{1}{2}b \cdot h$$

Again, the height must be perpendicular to the base.

3. Area of a Trapezoid

This is the specific formula students forget most often.

The Trapezoid Formula

$$A = \frac{b_1 + b_2}{2} \cdot h$$

How to memorize it:
You are taking the **Average of the two bases** and multiplying by the height.

Worked Example

A trapezoid has a top base of 6, a bottom base of 10, and a height of 4.

1. **Average the Bases:** $(6 + 10)/2 = 8$.
2. **Multiply by Height:** $8 \times 4 = 32$.

4. Volume: Prisms and Cylinders

Volume is the space inside a 3D object.
The General Rule: Area of the Base $\times$ the Height.

A. Rectangular Prism (Box)

$$V = l \cdot w \cdot h$$

Think: Area of floor $(l \cdot w)$ times the height of the room (h).

B. Cylinder (Can)

This is just a circle extended upwards.

$$V = \pi r^2 h$$

- πr^2 = Area of the circular base.
- h = Height of the cylinder.

Pro Tip: "In Terms of Pi"

If the answers look like 50π, 100π, etc., do **not** multiply by 3.14. Treat π like a variable (x).

Practice Problems

1. **Triangle Area:** A triangle has a base of 10 and a height of 5. What is its area?
2. **Trapezoid Area:** A trapezoid has parallel sides of length 4 and 12. The distance between them (height) is 5. What is the area?
3. **Rectangle Perimeter:** A rectangle has an area of 24 and a width of 4. What is the perimeter?
4. **Cylinder Volume:** A soda can has a radius of 3 cm and a height of 10 cm. What is the volume?
5. **Composite Area:** A shape is made of a square (side 4) topped by a triangle with a height of 3. What is the total area?
6. **Finding Height:** A parallelogram has an area of 50 and a base of 10. What is the height?
7. **Cube Volume:** If a cube has a volume of 27, what is the length of one side?
8. **Trapezoid Algebra:** A trapezoid has an area of 30. The bases are 4 and 6. What is the height?
9. **Slant Height Trap:** A parallelogram has a base of 10. The slanted side is 5, but the perpendicular height is 4. What is the area?
10. **Garden Path:** A rectangular garden is 10x20. A 2-foot wide path surrounds it. What is the area of just the path?

Solutions & Explanations

1. Answer: 25
$A = \frac{1}{2}(10)(5) = 25$.

2. Answer: 40
Average bases: $(4 + 12)/2 = 8$. Multiply by height: $8 \times 5 = 40$.

3. Answer: 20
Find Length: $A = l \cdot w \Rightarrow 24 = l \cdot 4 \Rightarrow l = 6$.
Perimeter: $4 + 4 + 6 + 6 = 20$.

4. Answer: 90π
Base Area: $\pi(3^2) = 9\pi$.
Volume: $9\pi \times 10 = 90\pi$.

5. Answer: 22
Square Area: $4 \times 4 = 16$.
Triangle Area: Base is 4 (top of square). $A = \frac{1}{2}(4)(3) = 6$.
Total: $16 + 6 = 22$.

6. Answer: 5
$50 = 10 \cdot h \Rightarrow h = 5$.

7. Answer: 3
Cube volume is s^3. $s^3 = 27 \Rightarrow s = 3$.

8. Answer: 6
Average bases: $(4 + 6)/2 = 5$.
$A = (\text{Avg}) \cdot h \Rightarrow 30 = 5h \Rightarrow h = 6$.

9. Answer: 40
Ignore the slanted side (5). Use the perpendicular height (4).
$10 \times 4 = 40$.

10. Answer: 136
Inner Area (Garden): $10 \times 20 = 200$.
Outer Dimensions: Add 2 to *each* side. New Width = 10+2+2 = 14. New Length = 20+2+2 = 24.
Outer Area: $14 \times 24 = 336$.
Path Area: $336 - 200 = 136$.