ACT Math Guide: Circles

Summit Math Lab

Introduction

Circles appear in two different ways on the ACT: **Geometric Circles** (Area, Circumference, Angles) and **Algebraic Circles** (Equations on the coordinate plane).

This guide covers:

1. **Basic Formulas** (Area & Circumference)

2. **Sectors & Arcs** (Pieces of a circle)

3. **The Equation of a Circle** (The most common "stumper")

1. Geometric Basics

You generally need just two formulas here, but you must know when to use which.

- **Radius (r):** Distance from center to edge.

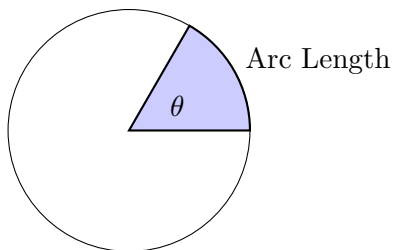- **Diameter (d):** Distance across the circle ($d = 2r$).

> **The Formulas**
>
> - **Area (Space Inside):** $A = \pi r^2$
>
> - **Circumference (Perimeter):** $C = 2\pi r$

Memory Tip: Area is measured in "square units," so it uses the **square** (r^2).

2. Sectors and Arc Length

Sometimes the ACT asks for just a *slice* of the pie (Sector Area) or just the *crust* (Arc Length). You find these by multiplying the Total Area or Circumference by a **Fraction**.

$$\text{Fraction} = \frac{\text{Central Angle}}{360°}$$

- **Sector Area:** $\frac{\theta}{360} \cdot \pi r^2$

- **Arc Length:** $\frac{\theta}{360} \cdot 2\pi r$

Worked Example

A circle has a radius of 6 and a central angle of $60°$. Find the arc length.

1. **Fraction:** $60/360 = 1/6$.

2. **Circumference:** $2\pi(6) = 12\pi$.

3. **Multiply:** $(1/6) \cdot 12\pi = 2\pi$.

3. The Equation of a Circle

This is the #1 concept students forget.

> **Standard Equation**
>
> $$(x - h)^2 + (y - k)^2 = r^2$$
>
> - **Center:** (h, k)
>
> - **Radius:** r

Crucial Warning: Flip the Signs!

The coordinates of the center are the **opposite** of what you see in the equation.

- Equation: $(x - 3)^2 + (y + 4)^2 = 25$

- **Center:** $(3, -4)$ (Positive 3, Negative 4)

- **Radius:** $\sqrt{25} = 5$ (Don't forget to square root!)

4. Advanced Property: Tangent Lines

A **tangent line** touches the circle at exactly one point.
The Rule: A tangent line is always **perpendicular** ($90°$) to the radius at the point of contact. This often creates a Right Triangle.

Practice Problems

1. **Basic Area:** If the circumference of a circle is 10π, what is its area?

2. **Equation Analysis:** What are the center and radius of $(x-4)^2 + (y+3)^2 = 16$?

3. **Writing Equations:** Write the equation of a circle with a center at $(-2, 0)$ and a radius of 3.

4. **Arc Length:** A circle has a radius of 9. What is the length of an arc defined by a $40°$ central angle?

5. **Sector Area:** A pizza has a diameter of 12 inches. If it is cut into 8 equal slices, what is the area of one slice?

6. **Diameter Endpoints:** A circle has a diameter with endpoints at $(1, 1)$ and $(7, 9)$. What is the equation of the circle?

7. **Inscribed Square:** A square is inscribed in a circle with radius 4. What is the area of the square?

8. **Tangent Line:** A point P is outside a circle. The distance from P to the center is 13. The distance from P to the point of tangency is 12. What is the radius?

9. **Completing the Square:** What is the center of the circle $x^2 + y^2 + 6x - 4y = 3$?

10. **Translation:** The circle $(x+1)^2 + y^2 = 9$ is shifted 3 units right and 2 units down. What is the new equation?

Solutions & Explanations

1. Answer: 25π
$C = 2\pi r = 10\pi \Rightarrow r = 5$. Area $= \pi(5)^2 = 25\pi$.

2. Answer: Center $(4, -3)$**, Radius** 4
Flip signs for center. Square root 16 for radius.

3. Answer: $(x+2)^2 + y^2 = 9$
Center $(-2, 0) \Rightarrow (x+2)^2 + (y-0)^2$. Radius $3 \Rightarrow 3^2 = 9$.

4. Answer: 2π
Fraction: $40/360 = 1/9$. $C = 18\pi$. Arc $= (1/9) \cdot 18\pi = 2\pi$.

5. Answer: 4.5π
Radius $= 6$. Area $= 36\pi$. Slice $= 36\pi/8 = 4.5\pi$.

6. Answer: $(x-4)^2 + (y-5)^2 = 25$
Midpoint (Center): $(\frac{1+7}{2}, \frac{1+9}{2}) = (4, 5)$.
Distance (Diameter): $\sqrt{(7-1)^2 + (9-1)^2} = \sqrt{36+64} = 10$. Radius $= 5$.

7. Answer: 32
Square diagonal $=$ Diameter $= 8$. Area of square $= d^2/2 = 64/2 = 32$.

8. Answer: 5
Tangent creates right triangle. Leg=12, Hyp=13. $r^2 + 12^2 = 13^2 \Rightarrow r = 5$.

9. Answer: $(-3, 2)$
Group: $(x^2 + 6x) + (y^2 - 4y) = 3$.
Complete square: $(x+3)^2 + (y-2)^2 = 3 + 9 + 4 = 16$. Center is $(-3, 2)$.

10. Answer: $(x-2)^2 + (y+2)^2 = 9$
Old Center: $(-1, 0)$. Shift $(+3, -2) \Rightarrow$ New Center $(2, -2)$.