ACT Math Guide: Exponents & Radicals

Summit Math Lab

Introduction

Exponents and Radicals (roots) are two sides of the same coin. The ACT loves to test your ability to switch between them. You must master the **5 Laws of Exponents** and the **Fractional Exponent Rule** to succeed.

This guide covers:

1. **The 5 Laws of Exponents** (Multiplying, Dividing, Powers)
2. **Negative & Zero Exponents** (The "Flip" and the "One")
3. **Fractional Exponents** (Converting to Radicals)
4. **Simplifying Radicals** (Breaking down square roots)

1. The 5 Laws of Exponents

When bases are the same (like x and x), you can combine them using these rules.

The Rules

1. **Product Rule:** $x^a \cdot x^b = x^{a+b}$ (Add powers)
2. **Quotient Rule:** $\frac{x^a}{x^b} = x^{a-b}$ (Subtract powers)
3. **Power Rule:** $(x^a)^b = x^{a \cdot b}$ (Multiply powers)
4. **Power of a Product:** $(xy)^a = x^a y^a$ (Distribute power)

Common Mistake: Adding vs. Multiplying

Students often confuse Rules 1 and 3.

- $x^2 \cdot x^3 = x^5$ (Think: $xx \cdot xxx = xxxxx$)
- $(x^2)^3 = x^6$ (Think: $x^2 \cdot x^2 \cdot x^2$)

2. Negative & Zero Exponents

These are conceptual rules that appear on almost every test.

A. The Zero Exponent ($x^0 = 1$)

Anything raised to the power of zero is **1**.
Exception: 0^0 is undefined.
Example: $500^0 = 1$, $(-5)^0 = 1$, $(xyz)^0 = 1$.

B. Negative Exponents ($x^{-n} = \frac{1}{x^n}$)

A negative exponent does **not** make the number negative. It creates a reciprocal (it flips the fraction).
Example: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.
Example: $\frac{1}{x^{-4}} = x^4$ (It moves from bottom to top).

3. Fractional Exponents

This is the bridge between Exponents and Radicals. You need to know how to rewrite them.

Power over Root

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

- Top number (a) = **Power** (stays with x)
- Bottom number (b) = **Root** (goes outside)

Worked Example

Evaluate $8^{\frac{2}{3}}$.
Step 1: Convert. The 3 is the root (cube root). The 2 is the power.

$$\sqrt[3]{8^2} \quad \text{OR} \quad (\sqrt[3]{8})^2$$

Step 2: Solve. It is usually easier to do the root first.
Cube root of 8 is 2.
$2^2 = 4$.
Answer: 4.

4. Simplifying Radicals

The ACT rarely leaves answers as $\sqrt{50}$. You must simplify by finding perfect square factors.

Perfect Squares: 4, 9, 16, 25, 36, 49, 64, 81, 100.

Worked Example

Simplify $\sqrt{72}$.
Step 1: Find the largest perfect square that divides 72.
(Factors: 2, 3, 4, 6, 8, 9, 12, 18, 24, **36**, 72).
Step 2: Split the radical.

$$\sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2}$$

Step 3: Solve the perfect square.

$$6\sqrt{2}$$

Practice Problems

1. **Basic Rules:** Simplify $(x^3)(x^4)$.
2. **Power Rule:** Simplify $(2x^2)^3$.
3. **Quotient Rule:** Simplify $\frac{x^8}{x^2}$.
4. **Zero Exponent:** Evaluate $5(4x)^0$.
5. **Negative Exponent:** What is the value of 4^{-2}?
6. **Fraction to Radical:** Rewrite $x^{3/4}$ in radical form.
7. **Evaluate Fractional:** What is the value of $27^{2/3}$?
8. **Simplify Radical:** Simplify $\sqrt{45}$.
9. **Radical Operations:** What is $\sqrt{20} + \sqrt{5}$? (*Hint: Simplify* $\sqrt{20}$ *first*).
10. **Solving Equations:** If $x^{1/2} = 6$, what is the value of x?

Solutions & Explanations

1. Answer: x^7
Product rule: Add exponents $(3 + 4)$.

2. Answer: $8x^6$
Distribute the cube to both the 2 and the x^2.
$2^3 = 8$. $(x^2)^3 = x^6$.

3. Answer: x^6
Quotient rule: Subtract exponents $(8 - 2)$.

4. Answer: 5
$(4x)^0$ becomes 1. So, $5(1) = 5$.
Warning: If it was $(20x)^0$, the answer would be 1. But the 5 is outside.

5. Answer: $\frac{1}{16}$
Negative flips it: $1/4^2 = 1/16$.

6. Answer: $\sqrt[4]{x^3}$
Top is Power (3), Bottom is Root (4).

7. Answer: 9
Rewrite: $(\sqrt[3]{27})^2$. Cube root of 27 is 3. $3^2 = 9$.

8. Answer: $3\sqrt{5}$
Factors of 45: 9×5. $\sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$.

9. Answer: $3\sqrt{5}$
Simplify $\sqrt{20}$ first: $\sqrt{4 \cdot 5} = 2\sqrt{5}$.
Add: $2\sqrt{5} + 1\sqrt{5} = 3\sqrt{5}$.

10. Answer: 36
$x^{1/2}$ means $\sqrt{x}$. So $\sqrt{x} = 6$. Square both sides: $x = 36$.