

ACT Math Guide: Functions (Notation & Translations)

Summit Math Lab

Introduction

Functions are simply "input/output machines." You put a number in (x), and a number comes out (y).

This topic intimidates students because of the notation, but it follows very strict rules. This guide covers:

1. **Basic Evaluation:** Plugging numbers in.
2. **Composite Functions:** The "Onion" method ($f(g(x))$).
3. **Graph Translations:** Moving the graph up, down, left, or right.

1. Function Notation: $f(x)$

$f(x)$ is just a fancy way of writing y .

- **Rule:** Whatever is inside the parentheses is your **Input**.
- **Rule:** The result after you do the math is your **Output**.

Worked Example

If $f(x) = x^2 - 4x$, find $f(-3)$.

Step 1: Identify the input is -3 .

Step 2: Replace *every* x with (-3) .

$$f(-3) = (-3)^2 - 4(-3)$$

Step 3: Simplify.

$$9 - (-12) = 9 + 12 = 21$$

Common Mistake: The "Phantom" Negative

When plugging in negative numbers, always use parentheses!

Wrong: $-3^2 - 12 \rightarrow -9 - 12 = -21$.

Right: $(-3)^2 - 12 \rightarrow 9 - 12 = -3$.

2. Composite Functions: $f(g(x))$

This is a function inside another function.

The Golden Rule: Work from the **Inside Out**.

Numerical Example

Find $f(g(4))$ if $f(x) = 2x$ and $g(x) = x + 5$.

1. **Inside First:** Find $g(4)$.
 $g(4) = 4 + 5 = 9$.
2. **Outside Second:** Plug the result (9) into $f(x)$.
 $f(9) = 2(9) = 18$.

Algebraic Example (The "Fog" Notation)

Sometimes written as $(f \circ g)(x)$, this means $f(g(x))$.

If $f(x) = x^2$ and $g(x) = x - 1$, find $f(g(x))$.

Action: Take the *entire* $g(x)$ equation and plug it into $f(x)$.

$$f(x - 1) = (x - 1)^2$$

(You can FOIL this out to $x^2 - 2x + 1$ if the answers require it).

3. Graph Translations

The ACT loves asking how an equation changes if you shift the graph.

The Rules of Movement

- **Outside (y):** Changes strictly to the end of the function move it **Vertically**. It follows your intuition.
- **Inside (x):** Changes inside the parentheses move it **Horizontally**. It is the **OPPOSITE** of your intuition.

Vertical Shifts (Outside)

- $y = f(x) + 2 \rightarrow$ Shift ****UP**** 2 units.
- $y = f(x) - 2 \rightarrow$ Shift ****DOWN**** 2 units.

Horizontal Shifts (Inside)

- $y = f(x + 2) \rightarrow$ Shift ****LEFT**** 2 units (Negative direction).

- $y = f(x - 2) \rightarrow$ Shift ****RIGHT**** 2 units (Positive direction).

Think: "To get back to zero, what do I need?" If you have $(x - 2)$, you need positive 2 to get back to zero. That's why it moves Right.

Practice Problems

1. **Basic Evaluation:** If $h(x) = -2x^2 + 5$, what is $h(-3)$?
 2. **Solving Backwards:** If $f(x) = 4x - 7$ and $f(x) = 13$, what is x ?
 3. **Composite (Numbers):** If $f(x) = x^2 + 1$ and $g(x) = 2x$, what is $f(g(3))$?
 4. **Composite (Reverse):** Using the functions above, what is $g(f(3))$?
 5. **Composite (Algebra):** If $f(x) = 3x$ and $g(x) = x^2 + 4$, what is $f(g(x))$?
 6. **Translation ID:** How does the graph of $y = (x - 5)^2 + 3$ compare to $y = x^2$?
A) Right 5, Up 3 B) Left 5, Up 3 C) Right 5, Down 3
 7. **Writing Translations:** The graph of $y = |x|$ is shifted 2 units to the **left** and 4 units **down**. What is the new equation?
 8. **Reading Graphs:** The graph of $f(x)$ passes through the point $(4, 10)$. What is the value of $f(4)$?
 9. **Nested Functions:** If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 9$, what is $f(g(4))$?
 10. **Logic Check:** If a function $f(x)$ is shifted to the right by h units, the new equation is $f(x - h)$. If it is shifted left, it is $f(x + h)$. True or False?
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Solutions & Explanations

1. Answer: -13

$$h(-3) = -2(-3)^2 + 5 = -2(9) + 5 = -18 + 5 = -13.$$

2. Answer: 5

$$\text{Set equation equal to 13: } 4x - 7 = 13 \rightarrow 4x = 20 \rightarrow x = 5.$$

3. Answer: 37

$$\text{Inside: } g(3) = 2(3) = 6.$$

$$\text{Outside: } f(6) = 6^2 + 1 = 36 + 1 = 37.$$

4. Answer: 20

$$\text{Inside: } f(3) = 3^2 + 1 = 10.$$

$$\text{Outside: } g(10) = 2(10) = 20.$$

Note: Order matters! $f(g(x))$ is rarely the same as $g(f(x))$.

5. Answer: $3x^2 + 12$

Plug $x^2 + 4$ into $f(x)$.

$$3(x^2 + 4) = 3x^2 + 12.$$

6. Answer: A (Right 5, Up 3)

Inside $(x - 5)$ is Opposite \rightarrow Right 5.

Outside $+3$ is Normal \rightarrow Up 3.

7. Answer: $y = |x + 2| - 4$

Left 2 means inside is $(x + 2)$. Down 4 means outside is -4 .

8. Answer: 10

Function notation is just (x, y) . If the point is $(4, 10)$, the input 4 gives output 10.

9. Answer: 5

$$\text{Inside: } g(4) = 4^2 + 9 = 16 + 9 = 25.$$

$$\text{Outside: } f(25) = \sqrt{25} = 5.$$

10. Answer: True

Inside shifts are opposite. Subtracting h moves right; adding h moves left.