ACT Math Guide: Linear Algebra & Systems

Summit Math Lab

Introduction

”Solving for *x*” is the most frequent skill tested on the ACT. While you have likely been doing this since 8th grade, the ACT tries to trick you with **Systems of Equations** (two equations, two variables) and **Special Solution Cases** (No Solution vs. Infinite Solutions).

NOTE: Remember the "solution" represents the coordinate of the intersection of the lines.

This guide covers:

1. **Systems of Equations** (Substitution vs. Elimination)
2. **Special Solution Cases** (Parallel Lines & Identical Lines)
3. **Modeling with Systems** (Word Problems)

1. Systems of Equations

A “system” is just a set of two lines. Solving the system means finding the **point where the two lines intersect** (x, y). There are two main ways to do this.

Method A: Substitution

Best used when: One variable is already isolated (e.g., $y = 3x + 1$).

1. Take the isolated expression $(3x + 1)$.
2. Plug it into the *other* equation where y used to be.
3. Solve for x, then plug back in to find y.

Method B: Elimination (Combination)

Best used when: Both equations are in Standard Form $(Ax + By = C)$ and lined up.

1. Multiply one (or both) equations by a number so that one variable cancels out when you add them.
2. Add the equations straight down.

Common Mistake: Forgetting to Distribute the Negative

When subtracting equations or substituting expressions like $-(3x-2)$, students often forget to distribute the negative sign.

- **Wrong:** $5x-(3x-2) \to 5x-3x-2$
- **Right:** $5x-(3x-2) \to 5x-3x+2$

Worked Example (Elimination)

Problem: Solve the system for y.

$$\begin{cases} 2x+3y=12 \\ 4x-y=10 \end{cases}$$

Step 1: Align coefficients.
Multiply the bottom equation by **3** to cancel the y's.

$$3(4x-y)=3(10) \to 12x-3y=30$$

Step 2: Add the equations.

$$\begin{array}{rl} 2x+3y=12 & \\ 12x-3y=30 & (+) \\ \hline 14x+0y=42 & \end{array}$$

Step 3: Solve for x.

$$14x=42 \Rightarrow x=3$$

Step 4: Plug back in to find y.

$$2(3)+3y=12 \Rightarrow 6+3y=12 \Rightarrow 3y=6 \Rightarrow y=2$$

Answer: $y=2$ (The solution is $(3,2)$).

2. Special Solution Cases

Sometimes, lines don't cross nicely at one point. The ACT loves these conceptual questions.

The Two Special Cases

Case A: No Solution ($\emptyset$)

- **Graphically:** The lines are **Parallel**. They never touch.
- **Algebraically:** Variables cancel, leaving a **FALSE** statement (e.g., $0 = 5$).
- **Condition:** Same Slope, Different Y-Intercept.

Case B: Infinite Solutions (∞)

- **Graphically:** The lines are **Identical**.
- **Algebraically:** Variables cancel, leaving a **TRUE** statement (e.g., $0 = 0$).
- **Condition:** Same Slope, Same Y-Intercept.

Worked Example

Problem: For what value of k does the system below have **no solution**?

$$\begin{cases} 3x + 4y = 10 \\ kx + 8y = 25 \end{cases}$$

Step 1: Understand the Condition.
"No solution" means parallel lines. Parallel lines have the **same slope**.

Step 2: Compare Ratios.
In Standard Form ($Ax + By = C$), the ratio of x-coefficients must match the ratio of y-coefficients.

$$\frac{3}{k} = \frac{4}{8}$$

Step 3: Solve the proportion.

$$4k = 3(8) \Rightarrow 4k = 24 \Rightarrow k = 6$$

Answer: $k = 6$.

Practice Problems

1. **Basic Elimination:** If $3x - 2y = 16$ and $2x + 2y = 14$, what is the value of x?
2. **Substitution:** If $y = 4x - 1$ and $2x + y = 11$, what is the value of x?
3. **Modeling:** A store sells regular notebooks for \$3 and fancy notebooks for \$5. If a student buys 10 notebooks total and spends \$38, how many fancy notebooks did they buy?
4. **No Solution Logic:** Which of the following systems has NO solution?
   A) $y = 3x + 2$ and $y = -3x + 2$
   B) $y = 3x + 2$ and $y = 3x + 5$
   C) $y = 3x + 2$ and $2y = 6x + 4$
5. **Manipulation:** If $5x + 3y = 10$, what is the value of $10x + 6y$?
6. **Substitution Practice:** Solve the system for y:
$$\begin{cases} x = 2y + 3 \\ 3x - 5y = 11 \end{cases}$$
7. **Infinite Solutions:** For what value of c does the system below have infinitely many solutions?
$$\begin{cases} 2x + y = 4 \\ 6x + 3y = c \end{cases}$$
8. **Elimination Shortcut:** If $2x + 3y = 17$ and $2x - 3y = 3$, what is the value of x? (Hint: You don't need to find y first).
9. **Coin Problem:** A jar contains 20 coins consisting only of quarters (\$0.25) and dimes (\$0.10). If the total value is \$3.50, how many quarters are in the jar?
10. **The "Add Them Up" Trick:** If $3x + 4y = 18$ and $4x + 3y = 17$, what is the value of $x + y$? (Hint: Do not solve for x and y separately).

Solutions & Explanations

1. Answer: 6
Solution: The y's are already opposites. Add equations: $(3x + 2x) = 16 + 14 \Rightarrow 5x = 30 \Rightarrow x = 6$.

2. Answer: 2
Solution: Substitute $(4x - 1)$ for y: $2x + (4x - 1) = 11 \Rightarrow 6x - 1 = 11 \Rightarrow 6x = 12 \Rightarrow x = 2$.

3. Answer: 4
Solution: Equations: $r + f = 10$ and $3r + 5f = 38$.
Multiply first eq by -3: $-3r - 3f = -30$.
Add to second eq: $2f = 8 \Rightarrow f = 4$.

4. Answer: B
Solution: No solution = Parallel lines (same slope, different intercept).
Choice B has slope 3 for both lines, but intercepts 2 and 5.

5. Answer: 20
Solution: Notice that $10x + 6y$ is exactly double $5x + 3y$.
$2(5x + 3y) = 2(10) = 20$.

6. Answer: $y = 2$
Solution: Substitute $(2y + 3)$ into the second equation for x.
$3(2y + 3) - 5y = 11 \Rightarrow 6y + 9 - 5y = 11 \Rightarrow y + 9 = 11 \Rightarrow y = 2$.

7. Answer: $c = 12$
Solution: "Infinite solutions" means the lines are identical.
Notice that the left side of the second equation $(6x + 3y)$ is exactly 3 times the first equation $(2x + y)$.
Therefore, the constant c must also be 3 times the constant 4.
$c = 3(4) = 12$.

8. Answer: $x = 5$
Solution: Add the two equations directly. The $3y$ and $-3y$ cancel out.
$(2x + 2x) = 17 + 3 \Rightarrow 4x = 20 \Rightarrow x = 5$.

9. Answer: 10 Quarters
Solution: Eq 1 (Quantity): $q + d = 20 \Rightarrow d = 20 - q$.
Eq 2 (Value): $0.25q + 0.10d = 3.50$. (Multiply by 100 to clear decimals: $25q + 10d = 350$).
Substitute d: $25q + 10(20 - q) = 350 \Rightarrow 25q + 200 - 10q = 350$.
$15q = 150 \Rightarrow q = 10$.

10. Answer: 5
Solution: Add the two equations together.
$(3x + 4x) + (4y + 3y) = 18 + 17 \Rightarrow 7x + 7y = 35$.
Divide the entire equation by 7: $x + y = 5$.