ACT Math Guide: Ratios, Proportions, & Percents

Summit Math Lab

Introduction

Modeling questions on the ACT often look like long paragraphs, but they usually boil down to comparing two things (Proportions) or tracking changes (Percents).

This guide covers:

1. **Setting up Proportions** (The "Labels" Strategy)

2. **Percents** (Translation & Percent Change)

3. **Variation** (Direct vs. Inverse)

1. Setting up Proportions

A proportion is just two fractions set equal to each other.

$$\frac{a}{b} = \frac{c}{d}$$

The Golden Rule: Match Your Labels

The most common mistake is flipping the fractions. If you put "Miles" on top on the left, you **must** put "Miles" on top on the right.

Example: A car travels 120 miles in 2 hours. At that rate, how far will it travel in 5 hours?

$$\frac{120 \text{ miles}}{2 \text{ hours}} = \frac{x \text{ miles}}{5 \text{ hours}}$$

Solve: Cross-multiply.
$120(5) = 2(x) \Rightarrow 600 = 2x \Rightarrow x = 300$ miles.

2. Percents

A. The Translation Trick

When a question is short, translate the words directly into math symbols.

- "is" $\rightarrow =$

- "of" $\rightarrow \times$ (multiply)

- "what" $\rightarrow x$ (variable)

- "%" $\rightarrow /100$ (decimal)

Example: "What number is 15% of 80?"
Translation: $x = 0.15 \times 80$.
$x = 12$.

B. Percent Change

The ACT loves to ask by what percentage something increased or decreased.

> **Percent Change Formula**
>
> $$\% \text{ Change} = \frac{\text{New} - \text{Old}}{\text{Old}} \times 100$$

Example: A shirt was \$40. Now it is \$30. What is the percent discount?
New = 30, Old = 40.
$\frac{30-40}{40} = \frac{-10}{40} = -0.25$.
Multiply by 100: -25%. (A 25% decrease).

3. Direct vs. Inverse Variation

This is a specific type of proportion vocabulary found on the ACT.

[Image of direct vs inverse variation graphs]

A. Direct Variation ("Directly Proportional")

As one variable goes UP, the other goes UP.

- **Equation:** $y = kx$ (or $\frac{y}{x} = k$)

- **Think:** Constant Ratio.

- **Example:** Hours worked vs. Money earned.

B. Inverse Variation ("Inversely Proportional")

As one variable goes UP, the other goes DOWN.

- **Equation:** $y = \frac{k}{x}$ (or $xy = k$)

- **Think:** Constant Product.

- **Example:** Speed vs. Time (to travel a fixed distance).

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Quick Test

- If y varies **directly** as x: Divide y by x. You should get the same number every time.

- If y varies **inversely** as x: Multiply y and x. You should get the same number every time.

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Practice Problems

1. **Basic Proportion:** A recipe calls for 3 cups of flour for every 4 cups of sugar. If you use 10 cups of sugar, how much flour do you need?

2. **Percent Translation:** 40 is 20% of what number?

3. **Percent Change:** Last year, a school had 500 students. This year, it has 550 students. What is the percent increase?

4. **Direct Variation:** If y varies directly as x, and $y = 12$ when $x = 4$, what is y when $x = 9$?

5. **Inverse Variation:** If y varies inversely as x, and $y = 10$ when $x = 6$, what is y when $x = 4$?

6. **Compound Percent:** A $100 jacket is discounted by 20%. The new price is then taxed at 10%. What is the final price?

7. **Ratio Sharing:** The angles of a triangle are in the ratio $1 : 2 : 3$. What is the measure of the largest angle?

8. **Map Scale:** On a map, 2 inches represents 15 miles. If two cities are 7 inches apart on the map, how many miles apart are they in real life?

9. **Logic Check:** If x varies inversely as y, what happens to y if x is doubled?
   A) y doubles   B) y is halved   C) y stays the same

10. **Algebraic Percent:** If 30% of x equals 60% of y, what is x in terms of y?

Solutions & Explanations

1. Answer: 7.5 cups
Setup: $\frac{\text{Flour}}{\text{Sugar}} \to \frac{3}{4} = \frac{x}{10}$.
$4x = 30 \Rightarrow x = 7.5$.

2. Answer: 200
Translation: $40 = 0.20 \times x$.
Divide by 0.20: $x = 40/0.20 = 200$.

3. Answer: 10%
Formula: $\frac{550-500}{500} = \frac{50}{500} = 0.10$.
Convert to percent: 10%.

4. Answer: 27
Direct means $y = kx$. Find k first: $12 = k(4) \Rightarrow k = 3$.
New equation: $y = 3x$. Plug in 9: $y = 3(9) = 27$.

5. Answer: 15
Inverse means $xy = k$. Find k: $6 \cdot 10 = 60$. So $k = 60$.
New scenario: $4 \cdot y = 60 \Rightarrow y = 15$.

6. Answer: \$88
Step 1 (Discount): $100 - (0.20 \times 100) = 80$.
Step 2 (Tax): Tax is on the **new** price (\$80), not the old one!
$80 + (0.10 \times 80) = 80 + 8 = 88$.

7. Answer: $90°$
Ratio $1 : 2 : 3$ means terms are $1x, 2x, 3x$.
Total angles in a triangle $= 180$.
$1x + 2x + 3x = 180 \Rightarrow 6x = 180 \Rightarrow x = 30$.
Largest angle is $3x = 3(30) = 90°$.

8. Answer: 52.5 miles
$\frac{2 \text{ in}}{15 \text{ mi}} = \frac{7 \text{ in}}{x \text{ mi}}$.
$2x = 105 \Rightarrow x = 52.5$.

9. Answer: B (Halved)
Inverse variation $(xy = k)$ is like a seesaw. To keep the product constant, if you multiply one side by 2, you must divide the other by 2.

10. Answer: $x = 2y$
Translation: $0.30x = 0.60y$.
Divide by 0.30: $x = \frac{0.60}{0.30}y \Rightarrow x = 2y$.