ACT Math Guide: Right Triangles & Trigonometry

Summit Math Lab

Introduction

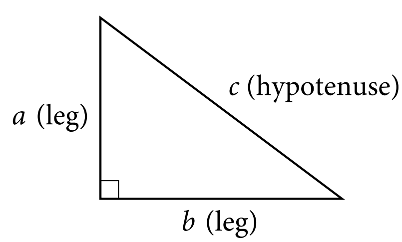
Trigonometry on the ACT can seem intimidating, but it is actually very predictable. While there are a few advanced questions, the vast majority of trig problems rely on just two concepts: **The Pythagorean Theorem** and **SOH CAH TOA**.

This guide covers:

1. **The Pythagorean Theorem** (Finding sides)
2. **SOH CAH TOA** (Relating angles and sides)
3. **Special Right Triangles** (The shortcuts)

1. The Pythagorean Theorem

This is the most fundamental rule of geometry. It applies **only** to right triangles (triangles with a 90° angle).



The Formula

$$a^2 + b^2 = c^2$$

Where c is the **hypotenuse** (the side opposite the right angle).

Common Mistake: Mixing up c

The hypotenuse (c) **must** be alone on one side of the equals sign. You cannot just add the squares of any two sides.
If you are looking for a leg: You must subtract. ($c^2 - a^2 = b^2$).

Worked Example

Problem: A right triangle has one leg of length 5 and a hypotenuse of length 13. What is the length of the other leg?

Step 1: Identify your variables.
We have a leg ($a = 5$) and the hypotenuse ($c = 13$). We are looking for b.

Step 2: Set up the equation.

$$5^2 + b^2 = 13^2$$

Step 3: Solve.

$$25 + b^2 = 169$$

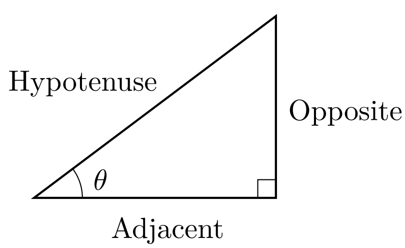
Subtract 25 from both sides:

$$b^2 = 144 \Rightarrow b = 12$$

Answer: The leg is **12**.

2. SOH CAH TOA (Trigonometric Ratios)

Trigonometry allows you to link **angles** to **side lengths**. To do this, you must be able to label the sides of the triangle relative to a specific angle (θ).



The Definitions

- **SOH:** $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$
- **CAH:** $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$
- **TOA:** $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

Common Mistake: "Opposite" vs "Adjacent"

"Opposite" and "Adjacent" depend on **which angle you are looking from**.

- **Opposite** is the side the angle is "looking at" (across the triangle).
- **Adjacent** is the side touching the angle (that isn't the hypotenuse).

Worked Example

Problem: In a right triangle, angle A is $30°$. The hypotenuse is 20. Find the length of the side opposite angle A.

Step 1: Choose the right ratio.
We have an angle ($30°$), Hypotenuse (20), and want Opposite (x). That is **SOH**.

Step 2: Set up the equation.

$$\sin(30°) = \frac{x}{20}$$

Step 3: Solve for x.

$$20 \cdot \sin(30°) = x$$

(Note: $\sin(30°) = 0.5$)

$$20(0.5) = x \Rightarrow x = 10$$

3. Pro Tip: Special Right Triangles

The ACT loves two specific types of triangles. If you memorize these ratios, you don't even need a calculator.

The Shortcuts

A. The 45-45-90 Triangle

- Legs are equal (x).
- Hypotenuse $= x\sqrt{2}$.

B. The 30-60-90 Triangle

- Short leg (opposite $30°$) $= x$
- Hypotenuse $= 2x$ (Double the short leg)
- Long leg (opposite $60°$) $= x\sqrt{3}$

Practice Problems

1. **The Ladder Problem:** A ladder is leaning against a wall. The base of the ladder is 6 feet away from the wall, and the ladder reaches 8 feet up the wall. How long is the ladder?
2. **Trig Setup:** In a right triangle, the side opposite angle θ is 7 and the adjacent side is 24. What is the value of $\cos(\theta)$?
3. **Solving for a Side:** A right triangle has an angle of 40°. The side adjacent to this angle is 10. Which expression represents the length of the opposite side?
   A) $10\tan(40°)$ B) $\frac{10}{\tan(40°)}$ C) $10\cos(40°)$
4. **Special Right Triangles:** In a 45°-45°-90° triangle, one leg is 6. What is the length of the hypotenuse?
5. **SOH CAH TOA Application:** A wire is attached to the top of a pole and staked into the ground. The wire makes a 60° angle with the ground. If the pole is 15 feet tall (opposite the angle), how long is the wire (hypotenuse)?

Solutions & Explanations

1. Answer: 10 feet
Solution: Use Pythagorean Theorem: $6^2 + 8^2 = c^2 \to 36 + 64 = 100$. $\sqrt{100} = 10$.

2. Answer: $\frac{24}{25}$
Solution: Find hypotenuse first: $7^2 + 24^2 = c^2 \to 49 + 576 = 625$. $\sqrt{625} = 25$.
$\cos(\theta) = \frac{\text{Adj}}{\text{Hyp}} = \frac{24}{25}$.

3. Answer: A $\mathbf{(}10\tan(40°)\mathbf{)}$
Solution: We have Adjacent (10) and want Opposite (x). Use TOA:
$\tan(40°) = \frac{x}{10} \Rightarrow x = 10\tan(40°)$.

4. Answer: $6\sqrt{2}$
Solution: In a 45-45-90, Hypotenuse = Leg $\cdot\sqrt{2}$. Since Leg = 6, Hypotenuse = $6\sqrt{2}$.

5. Answer: $10\sqrt{3}$
Solution: Use SOH: $\sin(60°) = \frac{15}{x}$.
$x = \frac{15}{\sin(60°)}$. Since $\sin(60°) = \frac{\sqrt{3}}{2}$, $x = \frac{15}{\frac{\sqrt{3}}{2}} = \frac{30}{\sqrt{3}}$.
Rationalize denominator: $\frac{30\sqrt{3}}{3} = 10\sqrt{3}$.