ACT Math Guide: Statistics & Probability

Summit Math Lab

Introduction

These questions usually appear at the very end of the test (Questions 50–60). They are often less computationally difficult than the earlier algebra questions, but because students are fatigued, they make careless errors.

This guide covers:

1. **The 4 Stats:** Mean, Median, Mode, Range.
2. **The "Sum Trick":** How to solve missing number problems.
3. **Weighted Averages:** When groups are different sizes.
4. **Probability:** Simple, Independent, and "Not" events.

1. The Big Four: Mean, Median, Mode, Range

A. Mean (Average)

$$\text{Mean} = \frac{\text{Sum of all numbers}}{\text{Total count of numbers}}$$

ACT Tip: Think of the Mean as the "balance point" of the data.

B. Median (The Middle)

The number exactly in the middle of the set.
Crucial Step: You **MUST** put the numbers in order from least to greatest first.

- **Odd set (e.g., 5 numbers):** The middle number is the answer.
- **Even set (e.g., 6 numbers):** There are two middles. Take the **average** of those two.

C. Mode (The Most Popular)

The number that appears most frequently.

- It is possible to have **No Mode** (all numbers appear once).
- It is possible to have **Multiple Modes** (a tie).

D. Range (The Spread)

$$\text{Range} = \text{Largest Number} - \text{Smallest Number}$$

Note: Range is a single number, not an interval. If the set is $\{2, 5, 10\}$, the range is 8 (because $10 - 2$), not "2 to 10".

2. Deep Dive: The "Sum Trick" (Missing Values)

This is the most common "Hard" statistics question.
Problem Type: "A student has an average of 80 on 4 tests. What does she need on the 5th test to get an 82 average?"

The Sum Formula

Since Average $= \frac{\text{Sum}}{\text{Count}}$, we can rearrange it:

$$\text{Sum} = \text{Average} \times \text{Count}$$

Worked Example: The Missing Test Score

Question: You have scores of 70, 80, and 90. What must you get on the 4th test to average an 85?

1. **Calculate the Desired Sum:** You want an average of 85 over 4 tests.

$$\text{Target Sum} = 85 \times 4 = 340$$

2. **Calculate Current Sum:** $70 + 80 + 90 = 240$.

3. **Find the Difference:** $340 - 240 = 100$.
   Answer: You need a 100.

3. Deep Dive: Weighted Averages

You cannot just average the averages if the groups are different sizes.

Worked Example: Uneven Groups

Question: Class A has 10 students with an average score of 90. Class B has 20 students with an average score of 80. What is the average of *both* classes combined?
Wrong Way: $(90 + 80)/2 = 85$. (This ignores that Class B is bigger).
Right Way (Total Sum / Total People):

1. **Sum of A:** 10 students $\times$ 90 = 900 points.

2. **Sum of B:** 20 students $\times$ 80 = 1600 points.

3. **Total Sum:** $900 + 1600 = 2500$.
4. **Total People:** $10 + 20 = 30$.
5. **Final Average:** $2500/30 = 83.3$.

4. Probability

A. Basic Probability

$$P(\text{Event}) = \frac{\text{Number of Desired Outcomes}}{\text{Total Number of Possible Outcomes}}$$

B. The "Not" Rule

The probability of something **NOT** happening is 1 minus the probability that it **does**.

$$P(\text{Not A}) = 1 - P(A)$$

C. Independent Events ("AND")

If you want Event A **AND** Event B to happen (and they don't affect each other), you **MULTIPLY**.
Example: Flipping a Heads AND rolling a 6.

$$P(A \text{ and } B) = P(A) \times P(B)$$

D. Mutually Exclusive Events ("OR")

If you want Event A **OR** Event B (and they can't happen at the same time), you **ADD**.
Example: Rolling a 2 OR a 4 on a single die.

$$P(A \text{ or } B) = P(A) + P(B)$$

Detailed Practice Problems

Statistics Problems

1. **Median Logic:** Find the median of the set $\{4, 12, 2, 9, 8, 20\}$.
2. **Missing Number:** The mean of a list of 5 numbers is 12. If four of the numbers are 8, 10, 15, and 12, what is the fifth number?
3. **Changed Mean:** A set of 4 numbers has a mean of 10. If the number 25 is added to the set, what is the new mean?
4. **Logic Check:** If a set of numbers contains only integers, is the mean always an integer?
5. **Weighted Average:** A basketball player scores 20 points in 3 games and 30 points in 2 games. What is his average score per game?

Probability Problems

6. **Basic Prob:** A jar contains 4 red, 5 blue, and 11 green marbles. What is the probability of drawing a blue marble?
7. **The "Not" Case:** In the jar above, what is the probability of drawing a marble that is **not** red?
8. **Multi-Step (AND):** You flip a coin and roll a standard 6-sided die. What is the probability of getting Tails and a number greater than 4?
9. **Expected Value (Hard):** A raffle has 100 tickets. One ticket wins \$100, and the rest win \$0. What is the expected value of one ticket?
10. **Geometric Probability:** A point is chosen at random inside a circle with radius 4. What is the probability that the point is also inside a smaller concentric circle with radius 2?

Solutions & Explanations

1. Answer: 8.5
Step 1: Order the list $\rightarrow \{2, 4, 8, 9, 12, 20\}$.
Step 2: Find the middle. Even set, so average 8 and 9.
$(8 + 9)/2 = 8.5$.

2. Answer: 15
Target Sum $= 12 \times 5 = 60$.
Current Sum $= 8 + 10 + 15 + 12 = 45$.
Missing Number $= 60 - 45 = 15$.

3. Answer: 13
Old Sum $= 10 \times 4 = 40$.
Add new number $= 40 + 25 = 65$.
New Count $= 5$ numbers.
New Mean $= 65/5 = 13$.

4. Answer: No
Counter-example: Set $\{1, 2\}$. Mean is 1.5.

5. Answer: 24
Sum 1: $20 \times 3 = 60$.
Sum 2: $30 \times 2 = 60$.

Total Sum $= 120$. Total Games $= 5$.
Average $= 120/5 = 24$.

6. Answer: 1/4 (or 25%)
Total marbles $= 4 + 5 + 11 = 20$.
Blue marbles $= 5$.
Prob $= 5/20 = 1/4$.

7. Answer: 4/5 (or 80%)
Method 1: Add Blue and Green $(5 + 11 = 16)$. Prob $= 16/20 = 4/5$.
Method 2 (Subtract): $1 - P(\text{Red}) = 1 - (4/20) = 1 - 1/5 = 4/5$.

8. Answer: 1/6
Prob(Tails) $= 1/2$.
Prob(> 4) includes 5 and 6 (2 numbers). Prob $= 2/6 = 1/3$.
Multiply: $(1/2) \times (1/3) = 1/6$.

9. Answer: $1
Sum of all winnings = \$100.
Total tickets = 100.
Average (Expected) Value $= 100/100 =$ \$1.

10. Answer: 1/4
This is an Area problem disguised as probability.
Total Area (Big Circle) $= \pi(4^2) = 16\pi$.
Desired Area (Small Circle) $= \pi(2^2) = 4\pi$.
Prob $= \frac{4\pi}{16\pi} = \frac{4}{16} = \frac{1}{4}$.