ACT Math Guide: The "Big Four" Coordinate Formulas

Summit Math Lab

Introduction

Coordinate Geometry makes up roughly **15–20% of the ACT Math section**. The good news? You can solve almost every single one of these questions if you master just four formulas.

The Catch: The ACT does **not** give you a formula sheet. You must memorize these.

This guide covers:

1. The Slope Formula
2. Slope-Intercept Form ($y = mx + b$)
3. The Distance Formula
4. The Midpoint Formula

1. The Slope Formula

The slope measures the "steepness" of a line. On the ACT, you will often be given two points and asked to find the slope of the line connecting them.

The Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Tip: Think of it as "Rise over Run." The change in y (vertical) goes on top; the change in x (horizontal) goes on the bottom.

Common Mistake: The Order Matters!

If you start with the y from the second point on top, you **must** start with the x from the second point on the bottom.

- **Correct:** $\frac{y_2 - y_1}{x_2 - x_1}$
- **Incorrect:** $\frac{y_2 - y_1}{x_1 - x_2}$ (This will give you the wrong sign!)

Worked Example

Problem: Find the slope of the line passing through the points $(2, -3)$ and $(5, 9)$.

Step 1: Label your points.
$(x_1, y_1) = (2, -3)$
$(x_2, y_2) = (5, 9)$

Step 2: Plug into the formula.

$$m = \frac{9 - (-3)}{5 - 2}$$

Step 3: Simplify.

$$m = \frac{9 + 3}{3} = \frac{12}{3} = 4$$

Answer: The slope is **4**.

2. Slope-Intercept Form ($y = mx + b$)

This is the most useful way to write the equation of a line because it tells you exactly what the line looks like instantly.

The Equation

$$y = mx + b$$

Where:

- m = Slope
- b = y-intercept (where the line crosses the vertical y-axis)

Crucial Concepts for ACT

- **Parallel Lines:** Have the **same** slope. (e.g., $y = 2x + 1$ and $y = 2x - 5$).
- **Perpendicular Lines:** Have **negative reciprocal** slopes. Flip the fraction and switch the sign. (e.g., If one slope is $\frac{2}{3}$, the perpendicular slope is $-\frac{3}{2}$).

Common Mistake: Forgetting to Isolate y

You cannot determine the slope just by looking at the number next to x if the equation is not in $y = \ldots$ form.
Example: In $2x + 3y = 9$, the slope is **not** 2. You must solve for y first ($y = -\frac{2}{3}x + 3$).

Worked Example

Problem: Find the equation of a line that is perpendicular to $y = -2x + 4$ and passes through the point $(0, 3)$.

Step 1: Find the new slope.
Original slope = -2. Perpendicular slope = $\frac{1}{2}$ (flip it and make it positive).

Step 2: Find the y-intercept (b).
The problem tells us the line passes through $(0, 3)$. Since the x-coordinate is 0, this **is** the y-intercept. So, $b = 3$.

Step 3: Write the equation.

$$y = \frac{1}{2}x + 3$$

3. The Distance Formula

This formula finds the straight-line distance between two points. It looks scary, but it is actually just the Pythagorean Theorem ($a^2 + b^2 = c^2$) in disguise.

The Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Common Mistake: Calculating Square Roots Too Early

Do **not** try to square root the individual terms. You must finish the addition inside the radical first.

- **Wrong:** $\sqrt{3^2 + 4^2} \to 3 + 4 = 7$
- **Right:** $\sqrt{3^2 + 4^2} \to \sqrt{9 + 16} \to \sqrt{25} = 5$

Worked Example

Problem: Find the distance between $(-1, 2)$ and $(2, 6)$.

Step 1: Plug points into the formula.

$$d = \sqrt{(2 - (-1))^2 + (6 - 2)^2}$$

Step 2: Simplify inside the parentheses.

$$d = \sqrt{(3)^2 + (4)^2}$$

Step 3: Square the numbers and add.

$$d = \sqrt{9 + 16} = \sqrt{25} = 5$$

4. The Midpoint Formula

This formula finds the exact center point between two coordinates. Think of it as finding the **average** of the x's and the **average** of the y's.

The Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Common Mistake: Subtracting Instead of Adding

In the Slope and Distance formulas, you subtract the numbers ($x_2 - x_1$). In Midpoint, you **add** them.
Tip to remember: If you want the average of two test grades, you add them up and divide by 2. You do the same thing here.

Worked Example

Problem: Find the midpoint of the segment connecting $(4, -8)$ and $(-2, 0)$.

Step 1: Plug into the formula.

$$x_{mid} = \frac{4 + (-2)}{2}, \quad y_{mid} = \frac{-8 + 0}{2}$$

Step 2: Simplify.

$$x_{mid} = \frac{2}{2} = 1, \quad y_{mid} = \frac{-8}{2} = -4$$

Answer: The midpoint is $(1, -4)$.

Practice Problems

1. **Slope Calculation:** What is the slope of the line passing through the points $(-3, 5)$ and $(1, -7)$?
2. **Identifying Slope:** What is the slope of the line given by the equation $4x - 2y = 10$?
3. **Parallel & Perpendicular:** Line A has the equation $y = 3x + 2$. Line B is perpendicular to Line A. What is the slope of Line B?
4. **Midpoint:** One endpoint of a line segment is $(2, 5)$ and the midpoint is $(5, 1)$. What are the coordinates of the other endpoint? (*Hint: Be careful! You are looking for an endpoint, not the midpoint.*)
5. **Distance:** What is the distance between the points $(1, 3)$ and $(7, 11)$?
6. **Equation of a Line:** Write the equation of a line that has a slope of -4 and passes through the point $(2, -1)$. Leave your answer in slope-intercept form ($y = mx + b$).

Solutions & Explanations

1. Answer: -3
Solution: Use the slope formula: $m = \frac{-7-5}{1-(-3)}$.
Simplify: $m = \frac{-12}{1+3} = \frac{-12}{4} = -3$.

2. Answer: 2
Solution: You must isolate y to put it in $y = mx + b$ form.
Subtract $4x$: $-2y = -4x + 10$.
Divide by -2: $y = 2x - 5$.
The number in front of x is 2.

3. Answer: $-\frac{1}{3}$
Solution: The slope of Line A is 3 (or $\frac{3}{1}$). Perpendicular lines have negative reciprocal slopes. Flip $\frac{3}{1}$ to $\frac{1}{3}$ and change the sign to negative.

4. Answer: $(8, -3)$
Solution: Let the missing endpoint be (x, y).
Set up the formula for x: $\frac{2+x}{2} = 5$. Multiply by 2: $2 + x = 10 \Rightarrow x = 8$.
Set up the formula for y: $\frac{5+y}{2} = 1$. Multiply by 2: $5 + y = 2 \Rightarrow y = -3$.

5. Answer: 10
Solution: Use distance formula: $d = \sqrt{(7-1)^2 + (11-3)^2}$.
Simplify: $d = \sqrt{6^2 + 8^2}$.
Square: $d = \sqrt{36 + 64} = \sqrt{100} = 10$.

6. Answer: $y = -4x + 7$
Solution: Start with $y = mx + b$. We know $m = -4$, so $y = -4x + b$.
Plug in the point $(2, -1)$ for x and y: $-1 = -4(2) + b$.
Solve: $-1 = -8 + b$. Add 8 to both sides: $7 = b$.
Final equation: $y = -4x + 7$.